SHIV SHAKTI

International Journal in Multidisciplinary and Academic Research (SSIJMAR)

Vol. 5, No. 5, October 2016 (ISSN 2278 – 5973)

A Study On The Relationship Between Rank Size Rule And Lognormal Rural Taluk Size Distribution

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Impact Factor = 3.133 (Scientific Journal Impact Factor Value for 2012 by Inno Space Scientific Journal Impact Factor)

Global Impact Factor (2013)= 0.326 (By GIF)

Indexing:

Abstract

The rural population of Tamil Nadu state is 34.9 million in 2001 and 37.2 million in 2011. The rural population census has the increasing tendency during 2001-2011 at Tamil Nadu state level. The growth of the rural population in Tamil Nadu state motivates to carry out the statistical study on the distribution of rural population at taluk level in Tamil Nadu state. Lognormal model is proposed to study the nature of the rural taluk size distribution of population using 2001 and 2011 population census data of Tamil Nadu state. Further as the empirical rural taluk size distribution are having skew in nature, lognormal model for rural taluk size distribution is proposed to establish the theoretical relationship between the lognormal model for Rural Taluk Size Distribution and strong expected rank size rule. Empirical evidences are necessary in support of the theoretical relationship between the lognormal Rural Taluk Size Distribution and strong expected rank size rule.

Introduction

Rural place located outside the city or town with population less than or equal to five thousand is called as a rural area. Classification of rural taluk size with respect to the size is called as Rural Taluk Size Distribution. It has the substantive interest in many socio and demographic fields. The General population in Tamil Nadu has been rapidly increasing due to natural growth. Established studies of population are reviewed critically and observed that A. Okabe (1979), B. Renganathan (1986, 2004) studied the urban population, the relationship between the rank size rule and the city size distribution analytically and empirically. Further, B. Renganathan (2005) studied the concentration of rural population in Tamil Nadu state using probabilistic model. The review of literature papers showed that none of the studies deals with Rural Taluk Size Distribution in Tamil Nadu State. The rural population census data of 2001 and 2011 were having an increasing tendency at Tamil Nadu level. The empirical distribution of rural taluk size distribution in both 2001 and 2011 census data showed the skew in nature. Hence it motivated to propose the lognormal model for rural taluk size is proposed to establish its theoretical relationship with strong expected rank size rule. Empirical evidences are necessary in support of the theoretical relationship between the lognormal model for rural taluk size and strong expected rank size rule. Hence the present investigation is undertaken.

Objective

To evaluate the relationship between the lognormal model and the strong expected rank size rule with the empirical evidence.

Data Source

Data on census 2001 and 2011 population (Directorate of census operations, Tamil Nadu, 2011) are applied for analyzing the nature of taluk size distribution in Tamil Nadu state.

Order statistics

The function $X_{(k)}$ of $(X_1, X_2, \ldots, X_n)$ that takes on the value $x_{(k)}$ in each possible sequence $(x_1, x_2, \ldots, x_n)$ of values assumed by $(X_1, X_2, \ldots, X_n)$ is known as the k^{th} order statistic (or) statistic of order k $\{X_{(1)}, X_{(2)}, \ldots, X_{(n)}\}$ is called the set of order statistics for $(X_1, X_2, \ldots, X_n)$.

Lognormal model

The basic underlying assumption of the lognormal model is “the law of proportionate effect”, or Gibrat’s law. Aitchison and Brown (1957) define the law as the process where “the change in the variate at any step in the process is a random proportion of the previous value of the variate”. That is, if the growth rate of a unit between time t_0 and t_1 is defined as,

$$g_1 = \frac{S_{t_1} - S_{t_0}}{S_{t_0}} \tag{1}$$

Then this number will be constant across all size classes. The growth rates are independent of size.

Aitchison and Brown (1957) show how Gibrat’s law generates a lognormal distribution. If we have the random variable S_t, then by (1)

$$\sum_{t=1}^{n} \frac{S_t - S_{t-1}}{S_{t-1}} = \sum_{t=1}^{n} g_t \tag{2}$$

and if the jump for each period is small.

$$\sum_{t=1}^{n} \frac{S_t - S_{t-1}}{S_{t-1}} \sim \int_{S_0}^{S_n} \frac{dS}{S} = \log S_n - \log S_0 \tag{3}$$

(1) and (3) ⇨

$$\text{Log } S_n = \log S_0 + g_1 + g_2 + \cdots + g_n \tag{4}$$

By the central limit theorem, the sum of independent random variables is asymptotically normal. Hence, the g’s are normally distributed and S_n is lognormally distributed. Thus, Gibrat’s law implies that growth rate is independent of size; the resulting distribution of size of rural taluks will be lognormal.

Let X be a random variable representing rural taluk size and it has the lognormal probability density function as,

$$f(x) = \begin{cases} \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\log x - \mu)^2}{2\sigma^2}} & , x > 0, -\infty < \mu < \infty \text{ and } \sigma > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

The random variable Y=log x is normal random variable having mean μ and variance$\sigma^2$where, log x denotes the natural logarithm of the rural population of taluks.

The distribution function of the lognormal model is,

$$F_X(x) = \text{P}(\text{X} \leq x)$$

$$= \frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{lnx - \mu}{\sigma\sqrt{2}}\right)$$

whereerf denotes the error function associated with the normal distribution.

The estimates of $\mu and \sigma are$

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{n} log_e x_i$$

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} f(log_e x_i)^2}{N} - \left(\frac{\sum_{i=1}^{n} f\, log_e x_i}{N}\right)^2}$$

where 'N' denotes the total number of rural taluks and 'n' denotes the number of rural taluk classes.

Beta distribution

Let X be a random variable representing rural taluk size and it has the Beta one distribution with parameters (r, n-r+1), then its probability density function is stated as,

$$\text{f}(x) = \frac{1}{B(\text{r,n}-\text{r}+1)}(x)^{r-1}(1-x)^{n-r} \; ; B(\text{r}, \text{n} - \text{r} + 1) > 0, 0 < x < 1$$

where$\text{B}(\text{r}, \text{n} - \text{r} + 1)$is the Betafunction.

The mean and variance of the rural taluk size distribution are obtained as,

$$\text{E (X)} = \frac{\text{r}}{\text{n}+1},$$

$$\text{V (X)} = \frac{\text{r}(\text{n}-\text{r}+1)}{(\text{n}+1)^2(\text{n}+2)}$$

Distribution of r^{th} order statistics

Let $X_{(r)}$be the r^{th} order statistics, then its probability density function is stated as,

$$g_r(x_r) = \frac{n!}{(r-1)!(n-r)!} [F(x_r)]^{r-1} [1-F(x_r)]^{n-r} f(x_r)$$

where F is the common distribution function of X.

Rank Size Rule

The relation,

$X_{(r)} R_{(r)}^{q} = C$, for all r = 1, 2, 3 . . . n,

where n is the no. of taluks,

$X_{(r)}$ is the size of the r^{th} ranked taluks,

$R_{(r)}$ is the rank of the r^{th} taluks, C and q are constants.

is called as rank size rule.

Rank size rule has been described probabilistically through an application of order statistics to study the relationship between rank size rule and Rural Taluk Size Distribution.

At a certain given point of time, 'n' taluks in a state are ordered or ranked according to their

sizes $X = [X_{(1)}, X_{(2)}, \ldots X_{(n)}]$. A. Okabe (1979) assumed that the set of observed values of X consists of 'n' taluk size values. These values are sampled according to the same RTSDF (X). As the observed Taluk Size values are sampled, ranked 'n' Taluk Size values X = [$X_{(1)}$ $X_{(2)} \ldots X_{(r)} \ldots X_{(n)}$; $X_{(1)} < X_{(2)} < \ldots \ldots < X_{(n)}$] are probabilistic. Then expected Taluk Size is obtained as,

$$E(X) = \{E[X_{(1)}]\ E[X_{(2)}] \ldots E[X_{(r)}] \ldots [E[X_{(n)}]\}$$

By using the expected Taluk Size, the Rank Size Rule is recalled as expected Rank Size Rule.

i.e., $E[X_{(r)}]\ R_{(r)}^{q} = C$, for all r = 1, 2. . . n,

Strong Expected Rank Size rule

A. Okabe (1979) established Strong Expected Rank Size Rule as,

$$E[X_{r/n}]\ R(r) = C(n) \quad \text{when } r = r^*, r^*+1, \ldots, n$$

$$n = r^*+1, r^*+2, \ldots$$

where r^* is the minimum positive integer such that R(r) > 0, R(r) rank function, is dependent of n, and C (n) is a constant function of 'n'.

The relationship between Rural Taluk Size Distribution and the Strong expected rank size rule is explained in the following section.

Lognormal Rural Taluk Size Distribution and Strong expected rank size rule

Let X_1, X_2, X_3, …, X_n be the Rural Taluk Size random variable sampled from the distribution function F and rural taluk size are independent identically distributed with lognormal probability density function,

$$f(x) = \begin{cases} \frac{1}{x\,\sigma\sqrt{2\pi}} e^{-\frac{(\log x-\mu)^2}{2\sigma^2}} & , x>0, -\infty < \mu < \infty \text{ and } \sigma > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

when $X_{(r)}$ be the r^{th} order Statistics, its probability density function corresponding to $X_{(r)}$ is obtained as,

$$f[x_{(r)}] = \begin{cases} \frac{1}{X_{(r)}\sigma\sqrt{2\pi}} e^{-\frac{(\log X_{(r)}-\mu)^2}{2\sigma^2}} & , x_{(r)}>0, -\infty < \mu < \infty \text{ and } \sigma > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

The standardized lognormal variable,

$$\zeta = \frac{X_{(r)} - \mu_1'}{\sqrt{\mu_2}}$$

$$= \frac{e^{Z_{(r)}\sigma + \mu} - e^{\mu + \frac{\sigma^2}{2}}}{[e^{\sigma^2}(e^{\sigma^2}-1)\, e^{2\mu}]^{\frac{1}{2}}}$$

where μ_1' is the mean of the lognormal distribution and μ_2 is the variance of the lognormal distribution.

$$\mu_1' = e^{\mu + \frac{\sigma^2}{2}},$$

$$\mu_2 = e^{\sigma^2}(e^{\sigma^2} - 1)\, e^{2\mu}$$

$$Z_{(r)} = \frac{\log X_{(r)} - \mu}{\sigma}$$

⇨ $\text{Log } x_{(r)} = Z_{(r)}\sigma + \mu$

$$x_{(r)} = e^{Z_{(r)}\sigma + \mu}$$

$$\therefore \quad \zeta = \frac{e^{Z_{(r)}\sigma + \mu} - e^{\mu + \frac{\sigma^2}{2}}}{[e^{\sigma^2}(e^{\sigma^2}-1)\, e^{2\mu}]^{\frac{1}{2}}}$$

$$= \frac{e^{\mu}\,(e^{Z_{(r)}\sigma} - e^{\frac{\sigma^2}{2}})}{e^{\mu}\,[e^{\sigma^2}\left(e^{\sigma^2}-1\right)]^{\frac{1}{2}}}$$

$$= \frac{e^{Z_{(r)}\sigma} - e^{\frac{\sigma^2}{2}}}{[e^{\sigma^2}(e^{\sigma^2}-1)]^{\frac{1}{2}}} \rightarrow (1)$$

As $\sigma \rightarrow 0$,

$(1) \rightarrow Z_{(r)} = \frac{\log X_{(r)} - \mu}{\sigma} \rightarrow N(0, 1)$

i.e. $\zeta \rightarrow$ Unit normal (or) standardized normal distribution as $\sigma \rightarrow 0$.

i.e. Lognormal distribution gives a good approximation to normal distribution.

$$Z_{(r)} = \frac{\log X_{(r)} - \mu}{\sigma} \sim N(0, 1) \text{ as } n \rightarrow \infty$$

The probability density function of $Z_{(r)}$ is

$$f[z_{(r)}] = \frac{1}{\sqrt{2\pi}} e^{\frac{-z_{(r)}^2}{2}} \quad \text{when } -\infty < z_{(r)} < \infty$$

The distribution function of $Z_{(r)}$ is stated as,

$$F[z_{(r)}] = \int_{-\infty}^{Z_{(r)}} f[z_{(r)}]\,dz_{(r)}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z_{(r)}} e^{\frac{-z_{(r)}^2}{2}}\,dz_{(r)}$$

The r^{th} order Rural Taluk Size Distribution is stated as,

$$f[z_{(r)}] = \frac{1}{B(r, n-r+1)} f[z_{(r)}] [F(z_{(r)})]^{r-1} [1-F(z_{(r)})]^{n-r}$$

Let $F[z_{(r)}] = U_{(r)}$

The r^{th} order Rural Taluk Size Distribution is,

$$g[z_{(r)}] = \frac{1}{B(r, n-r+1)} f[z_{(r)}] [U_{(r)}]^{r-1} [1-U_{(r)}]^{n-r} \frac{1}{f[z_{(r)}]}$$

$$= \frac{1}{B(r, n-r+1)} [U_{(r)}]^{r-1} [1-U_{(r)}]^{n-r}, \quad 0 < U_{(r)} < 1$$

which is called as a beta one distribution with parameters (r, n-r+1)

$$E[U_{(r)}] = \frac{r}{r+n-r+1} = \frac{r}{n+1}$$

$$V[U_{(r)}] = \frac{r(n-r+1)}{(n+1)^2(r+n-r+1+1)}$$

$$=\frac{r(n-r+1)}{(n+1)^2)(n+2)}$$

$$E\,[U_{(r)}] = \frac{r}{n+1} = E\,[F\,(z_{(r)})]$$

By using the probability integral transformation,

$F^{-1}[U_{(r)}] = z_{(r)}$ $\quad\quad$ $[\because U_{(r)} = F\,(z_{(r)})]$

$z_{(r)} = F^{-1}\,[U_{(r)}]$

$E\,[Z_{(r)}] = E\,[F^{-1}(U_{(r)})]$

$\leq F^{-1}[E\,(U_{(r)})]$ $\quad\quad$ [$\because$ Jensen's inequality $f\,[E(X)] \leq E\,[f(x)]$

where, f is a convex and monotonic increasing function]

$$= F^{-1}\left(\frac{r}{n+1}\right)$$

$$=\frac{r}{n+1}$$

$$=\frac{g(r)}{n+1}\ , \quad\quad \text{where, } g(r) = r$$

$$\therefore E[Z_{(r)}] \leq \frac{g(r)}{n+1}.$$

when $F^{-1}[U_{(r)}]$ is linear function in $U_{(r)}$, $F^{-1}(\frac{1}{n+1})$ is linear, $g(r) = r$

$$E\,[Z_{(r)}] = \frac{r}{n+1}$$

$$E\,[Z_{(r)}].\frac{1}{r} = \frac{r}{n+1}$$

$E\,[Z_{(r)}]\,R(r) = c\,(n)$

where $R(r) = \frac{1}{r}$,

$$c\,(n) = \frac{1}{n+1}$$

The strong expected rank size rule,

$E\,[Z_{(r)}]\,R(r) = c\,(n)$

where $R(r) = \frac{1}{r}$,

$c(n) = \frac{1}{n+1}$

is satisfied by the Lognormal Rural Taluk Size Distribution because

$F^{-1}[U_{(r)}]$: inf { $Z_{(r)}$: F[$z_{(r)}$] $\geq \frac{r}{n+1}$ }

is satisfied by Lognormal Rural Taluk Size Variable.

Empirical results

Rural Taluk Size Distribution of General population based on 2001 and 2011 census resembles the skew distribution as seen in Table -1.

Table -1 Empirical distribution of rural taluk size-2001 and 2011

General population

Rural Taluk Size (in'000)	**Number of rural taluks**	
	2001	2011
0-55	7	12
55-110	39	42
110-165	64	58
165-220	36	40
220-275	21	20
275-330	22	19
330-385	7	6
> 385	4	9
Total	**200**	**206**

Lognormal Model

Lognormal Model is fitted using **2001** population data given in the Table - 1 as follows:

The estimates of the parameters μ and σ^2 in the normal distribution are obtained as,

$\hat{\mu}$ =5.0321, $\hat{\sigma}$ =0.5577

Based on $\hat{\mu}$ and$\hat{\sigma}$, mean and variance of the lognormal distribution are obtained as,

The mean of the empirical rural taluk size distribution,

$\bar{X}$ = 179.0383,

The variance of the empirical rural taluk size distribution,

S^2= 11696.6330

The fitted model is described as,

$$f(x) = \begin{cases} \frac{1}{x(0.5577)\sqrt{2\pi}} e^{\frac{-1}{2(0.5577)^2}(\log x - 5.0321)^2} & ; x>0 \\ 0 & ; \text{otherwise} \end{cases}$$

The fitted function f(x) of Rural Taluks is obtained and presented in Table - 2 below:

Table - 2 Observed and the expected distribution of rural taluks for

General Population -2001

Rural Taluk Size (in'000)	Observed number of rural taluks	Probability values f(x)
0-55	7	0.0561
55-110	39	0.243
110-165	64	0.2766
165-220	36	0.1889
220-275	21	0.1112
275-330	22	0.0627
330-385	7	0.0352
> 385	4	0.0493
Total	**200**	**1.0000**

* Lognormal Model is fitted using **2011** population data given in the Table -1 as follows:

The estimates of the parameters μ and σ^2in the normal distribution are obtained as,

$\hat{\mu}$ =4.9967, $\hat{\sigma}$ =0.6252

Based on $\hat{\mu}$ and$\hat{\sigma}$, mean and variance of the lognormal distribution are obtained as, The mean of the empirical rural taluk size Distribution,

$\bar{X}$ = 179.8458

The variance of the empirical rural taluk size distribution,

S^2 = 15467.4963

The fitted model is described as,

$$f(x) = \begin{cases} \frac{1}{x(0.6256)\sqrt{2\pi}} e^{\frac{-1}{2(0.6256)^2}(\log x - 4.9967)^2} & ; x > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

The fitted function f(x) of Rural Taluks is obtained and presented in table -3 below:

Table - 3 Observed and the expected distribution of rural taluks for General Population -2011

Rural Taluk Size (in'000)	Observed number of rural taluks	Probability values f(x)
0-55	12	0.0568
55-110	42	0.261
110-165	58	0.2515
165-220	40	0.1679
220-275	20	0.1018
275-330	19	0.0613
330-385	6	0.0367
> 385	9	0.0630
Total	**206**	**1.0000**

Lognormal Rural Taluk Size Distribution - General Population 2001

The r^{th} order lognormal distribution function F [$z_{(r)}$] values have been computed using the information given in Table -1 and presented in Table -4 below:

Table -4 Computation of lognormal F [$z_{(r)}$] values-General

Population 2001

Rural Taluk Size (in'000)	Rank of the rural taluk(r)	F[z]	1-F[z]=R(z)	$Z_{(r)}$	$F[z_{(r)}]$
0-55	1	0.0561	0.9439	0.0064	0.5025
55-110	2	0.2761	0.7239	0.0468	0.5187
110-165	3	0.5527	0.4473	0.1617	0.5642
165-220	4	0.7416	0.2584	0.3872	0.6507
220-275	5	0.8528	0.1472	0.7704	0.7795
275-330	6	0.9155	0.0845	1.3881	0.9174
330-385	7	0.9507	0.0493	2.3914	0.9916
385-440	8	1.0000	0	∞	1.0000
440 and over	9	1.0000	0	∞	1.0000

The probability integral transformation is verified empirically as follows.

$$F^{-1}[F(z_{(r)})]: F^{-1}[U_{(r)}] = \inf \{Z_{(r)}: F[z_{(r)}] \geq \frac{r}{n+1}\}$$

When r = 1, $\{Z_{(1)}: F[z_{(1)}] = 0.5025 \geq \frac{1}{10} = 0.1\}$

When r = 2, $\{Z_{(2)}: F[z_{(2)}] = 0.5187 \geq \frac{2}{10} = 0.2\}$

When r = 3, $\{Z_{(3)}: F[z_{(3)}] = 0.5642 \geq \frac{3}{10} = 0.3\}$

When r = 4, $\{Z_{(4)}: F[z_{(4)}] = 0.6507 \geq \frac{4}{10} = 0.4\}$

When r = 5, $\{Z_{(5)}: F[z_{(5)}] = 0.7795 \geq \frac{5}{10} = 0.5\}$

When r = 6, $\{Z_{(6)}: F[z_{(6)}] = 0.9174 \geq \frac{6}{10} = 0.6\}$

When r = 7, $\{Z_{(7)}: F[z_{(7)}] = 0.9916 \geq \frac{7}{10} = 0.7\}$

When r = 8, $\{Z_{(8)}: F[z_{(8)}] = 1.0000 \geq \frac{8}{10} = 0.8\}$

When r = 9, $\{Z_{(9)}: F[z_{(9)}] = 1.0000 \geq \frac{9}{10} = 0.9\}$

It implies that Strong expected rank size rule is satisfied by the lognormal distribution of rural taluk size.

Lognormal Rural Taluk Size Distribution - General Population 2011

The r^{th} order lognormal distribution function $F[z_{(r)}]$ values have been computed using the information given in Table no.1 and presented in Table -5 below:

Table -5 Computation of lognormal $F[z_{(r)}]$ values-General Population 2011

Rural Taluk Size (in'000)	Rank of the rural taluk(r)	F[z]	1-F[z]=R(z)	$Z_{(r)}$	$F[z_{(r)}]$
0-55	1	0.0568	0.9432	0.0065	0.5026
55-110	2	0.3178	0.6822	0.0543	0.5216
110-165	3	0.5693	0.4307	0.1746	0.5693
165-220	4	0.7372	0.2628	0.3973	0.6544
220-275	5	0.8390	0.1610	0.7626	0.7771
275-330	6	0.9003	0.0997	1.3390	0.9097
330-385	7	0.9370	0.0630	2.2605	0.9881
385-440	8	1.0000	0	∞	1.0000
440 and over	9	1.0000	0	∞	1.0000

The probability integral transformation is verified empirically as follows.

$$F^{-1}[F(z_{(r)})]: F^{-1}[U_{(r)}] = \inf\{Z_{(r)}: F[z_{(r)}] \geq \frac{r}{n+1}\}$$

When r = 1, $\{Z_{(1)}: F[z_{(1)}] = 0.5026 \geq \frac{1}{10} = 0.1$

When r = 2, $\{Z_{(2)}: F[z_{(2)}] = 0.5216 \geq \frac{2}{10} = 0.2\}$

When r = 3, $\{Z_{(3)}: F[z_{(3)}] = 0.5693 \geq \frac{3}{10} = 0.3\}$

When r = 4, $\{Z_{(4)}: F[z_{(4)}] = 0.6544 \geq \frac{4}{10} = 0.4\}$

When r = 5, $\{Z_{(5)}: F[z_{(5)}] = 0.7771 \geq \frac{5}{10} = 0.5\}$

When r = 6, $\{Z_{(6)}: F[z_{(6)}] = 0.9097 \geq \frac{6}{10} = 0.6\}$

When r = 7, $\{Z_{(7)}: F[z_{(7)}] = 0.9881 \geq \frac{7}{10} = 0.7\}$

When r = 8, $\{Z_{(8)}: F[z_{(8)}] = 1.0000 \geq \frac{8}{10} = 0.8\}$

When r = 9, $\{Z_{(9)}: F[z_{(9)}] = 1.0000 \geq \frac{9}{10} = 0.9\}$

It implies that Strong expected rank size rule is satisfied by the lognormal distribution of rural taluk size.

Conclusion

The lognormal model is fitted to the empirical rural taluk size distribution based on the 2001 and 2011 census data for General population. A Rural Taluk Size Distribution is related to Rank Size Rule in terms of Strong expected Rank Size rule. It is shown that Strong expected Rank Size rule is satisfied by Lognormal Rural Taluk Size Distribution. Empirically proved that the lognormal model for Rural Taluk Size Distribution satisfy the strong expected rank size rule in both the years 2001 and 2011. Lognormal model confirm the real distribution of rural taluk size. The present investigation suggests to the future researchers for analyzing the nature of rural taluk size in all states of India.

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