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A Study On The Relationship Between Rank Size Rule And Exponential Rural Taluk Size Distribution

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Abstract

The rural population of Tamil Nadu state is 34.9 million in 2001 and 37.2 million in 2011. The rural population census has the increasing tendency during 2001-2011 at Tamil Nadu state level. The growth of the rural population in Tamil Nadu state motivates to carry out the statistical study on the distribution of rural population at taluk level in Tamil Nadu state. Exponential model is proposed to study the nature of the rural taluk size distribution of population using 2001 and 2011 population census data of Tamil Nadu state. Further as the empirical rural taluk size distribution are having skew in nature, Exponential model for rural taluk size distribution is proposed to establish the theoretical relationship between the lognormal model for Rural Taluk Size Distribution and strong expected rank size rule. Empirical evidences are necessary in support of the theoretical relationship between the Exponential Rural Taluk Size Distribution and strong expected rank size rule.

Introduction

Rural place located outside the city or town with population less than or equal to five thousand is called as a rural area. Classification of rural taluk size with respect to the size is called as Rural Taluk Size Distribution. It has the substantive interest in many socio and demographic fields. The General population in Tamil Nadu has been rapidly increasing due to natural growth. Established studies of population are reviewed critically and observed that A. Okabe (1979), B. Renganathan (1986, 2004) studied the urban population, the relationship between the rank size rule and the city size distribution analytically and empirically. Further, B. Renganathan (2005) studied the concentration of rural population in Tamil Nadu state using probabilistic model. The review of literature papers showed that none of the studies deals with Rural Taluk Size Distribution in Tamil Nadu State. The rural population census data of 2001 and 2011 were having an increasing tendency at Tamil Nadu level. The empirical distribution of rural taluk size distribution in both 2001 and 2011 census data showed the skew in nature. Hence it

motivated to propose the Exponential model for rural taluk size is proposed to establish its theoretical relationship with strong expected rank size rule. Empirical evidences are necessary in support of the theoretical relationship between the Exponential model for rural taluk size and strong expected rank size rule. Hence the present investigation is undertaken.

Objective

To evaluate the relationship between the Exponential model and the strong expected rank size rule with the empirical evidence.

Data Source

Data on census 2001 and 2011 population (Directorate of census operations, Tamil Nadu, 2011) are applied for analyzing the nature of taluk size distribution in Tamil Nadu state.

Order statistics

The function $X_{(k)}$ of $(X_1, X_2, \ldots, X_n)$ that takes on the value $x_{(k)}$ in each possible sequence $(x_1, x_2, \ldots, x_n)$ of values assumed by $(X_1, X_2, \ldots, X_n)$ is known as the k^{th} order statistic (or) statistic of order k $\{X_{(1)}, X_{(2)}, \ldots, X_{(n)}\}$ is called the set of order statistics for $(X_1, X_2, \ldots, X_n)$.

Exponential model

Let X be a random variable representing rural taluk size and it has the probability density function as,

$$f(x) = \begin{cases} \lambda e^{-\lambda x}; & x>0, \\ 0 \quad ; \text{otherwise} \end{cases}$$

where $\lambda>0$ is the parameter of the distribution and is estimated by the method of moments.

The distribution function of the exponential model is,

$$F_X(x) = P(X \leq x)$$

$$= 1 - e^{-\lambda x} \quad \text{for all } x>0; \lambda>0$$

The estimates of the parameter λ of the exponential model is obtained as,

$$\hat{\lambda} = \frac{1}{\bar{X}}$$

Beta distribution

Let X be a random variable representing rural taluk size and it has the Beta one distribution with parameters (r, n-r+1), then its probability density function is stated as,

$$f(x) = \frac{1}{B(r,n-r+1)}(x)^{r-1}(1-x)^{n-r} \; ; B(r, n-r+1) > 0, 0 < x < 1$$

where$B(r, n-r+1)$is the Betafunction.

The mean and variance of the rural taluk size distribution are obtained as,

$$E(X) = \frac{r}{n+1},$$

$$V(X) = \frac{r(n-r+1)}{(n+1)^2(n+2)}$$

Distribution of r^{th} order statistics

Let $X_{(r)}$be the r^{th} order statistics, then its probability density function is stated as,

$$g_r(x_r) = \frac{n!}{(r-1)!(n-r)!} [F(x_r)]^{r-1} [1-F(x_r)]^{n-r} f(x_r)$$

where F is the common distribution function of X.

Rank Size Rule

The relation,

$X_{(r)}R_{(r)}^{q} = C$, for all r = 1, 2, 3 . . . n,

where n is the no. of taluks,

$X_{(r)}$is the size of the r^{th} ranked taluks,

$R_{(r)}$is the rank of the r^{th}taluks, C and q are constants.

is called as rank size rule.

Rank size rule has been described probabilistically through an application of order statistics to study the relationship between rank size rule and Rural Taluk Size Distribution.

At a certain given point of time, 'n' taluks in a state are ordered or ranked according to

their sizes $X = [X_{(1)}, X_{(2)}, \ldots X_{(n)}]$. A. Okabe (1979) assumed that the set of observed values of X consists of 'n' taluk size values. These values are sampled according to the same RTSDF (X).As the observed Taluk Size values are sampled, ranked 'n' Taluk Size values $X = [X_{(1)} X_{(2)} \ldots X_{(r)} \ldots X_{(n)} ; X_{(1)} < X_{(2)} < \ldots \ldots < X_{(n)}]$ are probabilistic. Then expected Taluk Size is obtained as,

$$E(X) = \{E[X_{(1)}] E[X_{(2)}] \ldots E[X_{(r)}] \ldots [E[X_{(n)}]\}$$

By using the expected Taluk Size, the Rank Size Rule is recalled as expected Rank Size Rule.

i.e., $E[X_{(r)}] R_{(r)}^{q} = C$, for all r = 1, 2. . . n,

Strong Expected Rank Size rule

A. Okabe (1979) established Strong Expected Rank Size Rule as,

$E[X_{r/n}] R(r) = C(n)$ when $r = r^*, r^*+1, \ldots, n$

$$n = r^*+1, r^*+2, \ldots$$

where r^* is the minimum positive integer such that R(r) > 0, R(r) rank function, is dependent of n, and C (n) is a constant function of 'n'.

The relationship between Rural Taluk Size Distribution and the Strong expected rank size rule is explained in the following section.

Exponential Rural Taluk Size Distribution and Strong expected rank size rule

Let $X_1, X_2, X_3, \ldots, X_n$ be the rural taluk size random variable sampled from the distribution function F and rural taluk size are independent identically distributed with exponential probability density function,

$$f(x) = \begin{cases} \lambda e^{-\lambda x} ; x>0, \lambda>0 \\ 0 \quad ; \text{otherwise} \end{cases}$$

when $X_{(r)}$ be the r^{th} order Statistics, its probability density function corresponding to $X_{(r)}$ is obtained as,

$$f[x_{(r)}] = \begin{cases} \lambda e^{-\lambda X_{(r)}}; \quad x_{(r)}>0, \lambda>0 \\ 0 \quad ; \text{otherwise} \end{cases}$$

The distribution function of $X_{(r)}$ is stated as,

$$F[x_{(r)}] = \int_0^{X_{(r)}} f[x_{(r)}]\, dx_{(r)}$$

$$= \int_0^{X_{(r)}} \lambda e^{-\lambda X_{(r)}}\, dx_{(r)}$$

The r^{th}order Rural Taluk Size Distribution is stated as,

$$f[x_{(r)}] = \frac{1}{B(r,n-r+1)} f[x_{(r)}] [F(x_{(r)})]^{r-1} [1-F(x_{(r)})]^{n-r}$$

Let $F[x_{(r)}] = U_{(r)}$

The r^{th}order Rural Taluk Size Distribution is,

$$g[x_{(r)}] = \frac{1}{B(r,n-r+1)} f[x_{(r)}] [U_{(r)}]^{r-1} [1-U_{(r)}]^{n-r} \frac{1}{f[X_{(r)}]}$$

$$= \frac{1}{B(r,n-r+1)} [U_{(r)}]^{r-1} [1-U_{(r)}]^{n-r}, \quad 0 < U_{(r)} < 1$$

This is called as a beta one distribution with parameters (r, n-r+1)

$$E[U_{(r)}] = \frac{r}{r+n-r+1} = \frac{r}{n+1}$$

$$V[U_{(r)}] = \frac{r(n-r+1)}{(n+1)^2(r+n-r+1+1)}$$

$$= \frac{r(n-r+1)}{(n+1)^2)(n+2)}$$

$$E[U_{(r)}] = \frac{r}{n+1} = E[F[x_{(r)}]] \qquad [\because U_{(r)} = F[x_{(r)}]]$$

By using the probability integral transformation,

$$F^{-1}[U_{(r)}] = x_{(r)} \qquad [\because U_{(r)} = F[x_{(r)}]]$$

$$x_{(r)} = F^{-1}[U_{(r)}]$$

$$E[X_{(r)}] = E[F^{-1}(U_{(r)})]$$

$$\leq F^{-1}[E(U_{(r)})] \qquad [\because \text{Jensen's inequality } f[E(X)] \leq E[f(x)]$$

where, f is a convex and monotonic increasing function]

$$= F^{-1}\left(\frac{r}{n+1}\right)$$

$$= \frac{r}{n+1}$$

$$= \frac{g(r)}{n+1}, \qquad \text{where, } g(r) = r$$

$\therefore E[X_{(r)}] \leq \frac{g(r)}{n+1}$.

when $F^{-1}[U_{(r)}]$ is linear function in $U_{(r)}$, $F^{-1}\left(\frac{1}{n+1}\right)$ is linear, $g(r) = r$

$$E[X_{(r)}] = \frac{r}{n+1}$$

$$E[X_{(r)}].\frac{1}{r} = \frac{1}{n+1}$$

$$E[X_{(r)}]\, R(r) = c(n)$$

where $R(r) = \frac{1}{r}$,

$$c(n) = \frac{1}{n+1}$$

The strong expected rank size rule,

$$E[X_{(r)}]\, R(r) = c(n)$$

where $R(r) = \frac{1}{r}$,

$$c(n) = \frac{1}{n+1}$$

is satisfied by the Exponential Rural Taluk Size Distribution because

$$F^{-1}[U_{(r)}]: \inf\{ X_{(r)}: F[x_{(r)}] \geq \frac{r}{n+1} \}$$

is satisfied by Exponential Rural Taluk Size Variable.

Thus, the strong expected rank size rule has been satisfied by exponential Rural Taluk Size Distributions.

Empirical results

Rural Taluk Size Distribution of General population based on 2001 and 2011 census resembles the skew distribution as seen in Table -1.

Table -1 Empirical distribution of rural taluk size-2001 and 2011

General population

Rural Taluk Size (in'000)	Number of rural taluks	
	2001	2011
0-55	7	12
55-110	39	42
110-165	64	58
165-220	36	40
220-275	21	20
275-330	22	19
330-385	7	6
> 385	4	9

Total	**200**	**206**

Exponential Model

Exponential Model is fitted using **2001** population data given in the Table -1 as follows:

The mean of the empirical rural taluk size distribution is obtained as,

$\bar{X}$ = 175.725,

The estimates of the parameter of the exponential model are obtained as,

$\hat{\lambda} = 0.0057$

The fitted model is stated as,

$$f(x) = \begin{cases} (0.0056)\, e^{(-0.0056)x} & ; x > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

The fitted function f(x) of Rural Taluks is obtained and presented in Table -2.

Table -2 Observed and the expected distribution of rural taluks for General Population -2001

Rural Taluk Size (in'000)	Observed number of rural taluks	Probability values f(x)
0-55	7	0.2691
55-110	39	0.1967
110-165	64	0.1437
165-220	36	0.1051
220-275	21	0.0768
275-330	22	0.0561
330-385	7	0.041
> 385	4	0.1115
Total	**200**	**1.0000**

***** Exponential Model is fitted using **2011** population data given in the Table -1 as follows:

The mean of the empirical rural taluk size distribution is obtained as,

$\bar{X}$ = 174.8786,

The estimates of the parameter of the exponential model are obtained as,

$\hat{\lambda} = 0.0057$

The fitted model is stated as,

$$f(x) = \begin{cases} (0.0057)\, e^{(-0.0057)x} & ; x > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

The fitted function f(x) of Rural Taluks is obtained and presented in Table -3 below:

Table - 3 Observed and the expected distribution of rural taluks for General Population -2011

Rural Taluk Size (in'000)	Observed number of rural taluks	Probability values f(x)	Expected number of rural taluks
0-55	12	0.2691	55
55-110	42	0.1967	40
110-165	58	0.1437	30
165-220	40	0.1051	22
220-275	20	0.0768	16
275-330	19	0.0561	12
330-385	6	0.0410	8
> 385	9	0.1115	23
Total	**206**	**1.0000**	**206**

Exponential Rural Taluk Size Distribution - General Population 2001

The r^{th} order exponential distribution function F $[x_{(r)}]$ values have been computed using the information given in Table -1 and presented in Table -4 below:

Table -4 Computation of Exponential $F[x_{(r)}]$ values-General Population 2001

Rural Taluk Size (in'000)	Rank of the rural taluk (r)	F[x]	1-F[x]=R(x)	$X_{(r)}$	$F[x_{(r)}]$
0-55	1	0.2691	0.7309	0.0392	0.2692
55-110	2	0.4658	0.5342	0.1288	0.5941
110-165	3	0.6095	0.3905	0.2855	0.8197
165-220	4	0.7146	0.2854	0.5363	0.9315
220-275	5	0.7914	0.2086	0.9281	0.9756
275-330	6	0.8475	0.1525	1.5550	0.9906
330-385	7	0.8885	0.1115	2.6518	0.9950
> 385	8	1.0000	0.0000	∞	1.0000

The probability integral transformation is verified empirically as follows.

$$F^{-1}[F(x_{(r)})]: F^{-1}[U_{(r)}] = \inf\{X_{(r)}: F[x_{(r)}] \geq \frac{r}{n+1}\}$$

When r = 1, $\{X_{(1)}: F[x_{(1)}] = 0.2692 \geq \frac{1}{9} = 0.1111\}$

When r = 2, $\{X_{(2)}: F[x_{(2)}] = 0.5941 \geq \frac{2}{9} = 0.2222\}$

When r = 3, $\{X_{(3)}: F[x_{(3)}] = 0.8197 \geq \frac{3}{9} = 0.3333\}$

When r = 4, $\{X_{(4)}: F[x_{(4)}] = 0.9315 \geq \frac{4}{9} = 0.4444\}$

When r = 5, $\{X_{(5)}: F[x_{(5)}] = 0.9756 \geq \frac{5}{9} = 0.5555\}$

When r = 6, $\{X_{(6)}: F[x_{(6)}] = 09906 \geq \frac{6}{9} = 0.6666\}$

When r = 7, $\{X_{(7)}: F[x_{(7)}] = 0.9950 \geq \frac{7}{9} = 0.7777\}$

When r = 8, $\{X_{(8)}: F[x_{(8)}] = 1.0000 \geq \frac{8}{9} = 0.8888\}$

It implies that Strong expected rank size rule is satisfied by the exponential distribution of rural taluk size.

Exponential Rural Taluk Size Distribution - General Population 2011

The r^{th} order exponential distribution function $F[x_{(r)}]$ values have been computed using the information given in Table -1 and presented in Table -5 below:

Table -5 Computation of Exponential $F[x_{(r)}]$ values-General Population 2011

Rural Taluk Size (in'000)	Rank of the rural taluk (r)	F[x]	1-F[x]=R(x)	$X_{(r)}$	$F[x_{(r)}]$
0-55	1	0.2691	0.7309	0.0392	0.2692
55-110	2	0.4658	0.5342	0.1288	0.5941
110-165	3	0.6095	0.3905	0.2855	0.8197
165-220	4	0.7146	0.2854	0.5363	0.9315
220-275	5	0.7914	0.2086	0.9281	0.9756

275-330	6	0.8475	0.1525	1.5550	0.9906
330-385	7	0.8885	0.1115	2.6519	0.9950
> 385	8	1.0000	0.0000	∞	1.0000

The probability integral transformation is verified empirically as follows.

$$F^{-1}[F(x_{(r)})]: F^{-1}[U_{(r)}] = \inf\{X_{(r)}: F[x_{(r)}] \geq \frac{r}{n+1}\}$$

When r = 1, $\{X_{(1)}: F[X_{(1)}] = 0.2692 \geq \frac{1}{9} = 0.1111\}$

When r = 2, $\{X_{(2)}: F[x_{(2)}] = 0.5941 \geq \frac{2}{9} = 0.2222\}$

When r = 3, $\{X_{(3)}: F[x_{(3)}] = 0.8197 \geq \frac{3}{9} = 0.3333\}$

When r = 4, $\{X_{(4)}: F[x_{(4)}] = 0.9315 \geq \frac{4}{9} = 0.4444\}$

When r = 5, $\{X_{(5)}: F[x_{(5)}] = 0.9756 \geq \frac{5}{9} = 0.5555\}$

When r = 6, $\{X_{(6)}: F[x_{(6)}] = 0.9906 \geq \frac{6}{9} = 0.6666\}$

When r = 7, $\{X_{(7)}: F[x_{(7)}] = 0.9950 \geq \frac{7}{9} = 0.7777\}$

When r = 8, $\{X_{(8)}: F[x_{(8)}] = 1.0000 \geq \frac{8}{9} = 0.8888\}$

It implies that Strong expected rank size rule is satisfied by the exponential distribution of rural taluk size.

Conclusion

The Exponential model is fitted to the empirical rural taluk size distribution based on the 2001 and 2011 census data for General population. A Rural Taluk Size Distribution is related to Rank Size Rule in terms of Strong expected Rank Size rule. It is shown that Strong expected Rank Size rule is satisfied by Exponential Rural Taluk Size Distribution. Empirically proved that the exponential model for Rural Taluk Size Distribution satisfy the strong expected rank size

rule. Exponential model confirm the real distribution of rural taluk size. The present investigation suggests to the future researchers for analyzing the nature of rural taluk size in all states of India.

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