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Stability Criteria Of Fully Connected Hopfield Artificial Neural Network

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Abstract:

This paper deals with a proposed model of realization of an interconnected Hopfield Artificial Neural Network (HANN). Hopfield Neural Network is a multiple loop feedback neural network which can be used as an associative memory. All the neurons in the network are connected to every other neuron but without any self-feedback. HANN can be used in Wireless Mesh Network (WMN). WMN adopts a multi-hop access method and expands network coverage by increasing the number of user-node and also enhances reliability of data transmission in both urban and rural areas. This paper is a humble attempt to derive the generic stability criteria of energy function for HANN which can be realised in WMN as well for better QoS.

Keywords: Neural Network,Neurons,Wireless Mesh Network

Introduction:

The ANN emulates the biological neural networks in that they do not require the programming of tasks but generalize and learn from experience. Current AN Networks comprise of a set of very simple processing elements that emulate the biological neurons and by a certain number of connections between them. They respond in parallel to the presented inputs, and can function correctly even through one of the processing units stops working or the information has a certain noise level. It's, therefore, a fault and noise tolerant system, able to learn from experiences and modifies the values associated with the processing unit-connections to adjust the output offered by the system in response to the inputs.

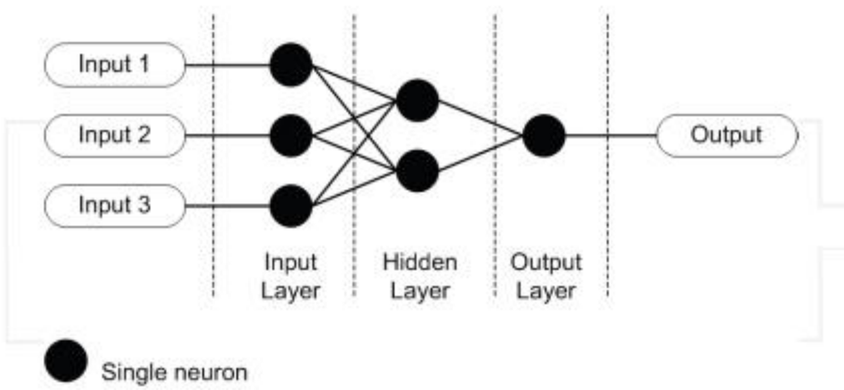

Fig 1: Simplified block diagram of a neural network

Realization of an artificial neuron with Transfer Function:

Artificial neurons are the building blocks of an ANN. In a biological neuron, it receives the input information through dendrites. Soma/ Cell body processes the information and eventually passes it to Axon. For an artificial neuron, weighted inputs come to the body of it which is summed up and finally processed by means of a TF (Transfer Function).

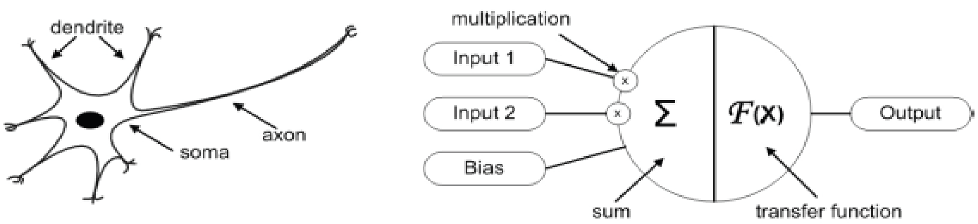

Fig 2: Comparative analogy of biological and artificial neurons

If y(k) be the output value at any discrete time (k), then, for an artificial neuron, the time-discrete output is given as:

$$y(k) = F\left(\sum_{i=0}^{m} w_i(k) \cdot x_i(k) + b\right)$$

Where,

$x_i(k)$ is the value of input;

$w_i(k)$: is value of the corresponding weight associated with the input

b : is corresponding bias &

F : is the transfer function

From this above equation, it's crystal clear that here we need to deal with one major unknown parameter and that is the transfer function which defines properties of every individual artificial neuron. It's chosen according to the requirement of the problem and can be any mathematical or stochastic function.

Introduction to HANN in WMN:

The Hopfield Artificial Neural Network is a dynamic and interconnected network, which iterates the converge from an arbitrary input state. It serves to minimize an energy function. It's a fully connected weighted network where the output of the network is fed back and there are weights associated to each of these links.

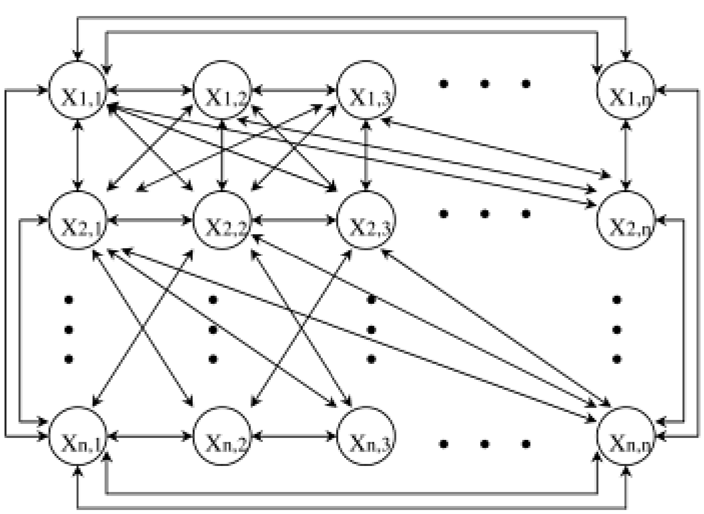

Fig3 :A typical fully connected Hopfield network

Here there are n ^{2} neurons in the network and n is the number of nodes. This kind of recurrent network is useful to store target vectors (Memories that a network recollects while it's provided similar data). The neurons have a threshold and step function. Every binary unit-linkage (a) or activation linkage assumes the values as defined below:

$$a_i = \begin{cases} -1 & \text{if } \sum_j w_{ij} s_j > \theta_i , \\ 1 & \text{otherwise.} \end{cases}$$

$$a_i = \begin{cases} 0 & \text{if } \sum_j w_{ij} s_j > \theta_i , \\ 1 & \text{otherwise.} \end{cases}$$

Where

w_{ij} is the strength of the connection weight from unit j to unit i,
s_j is the state of unit j,
θ_i is the threshold of unit i.

Equivalent Hardware realization and stability criteria:

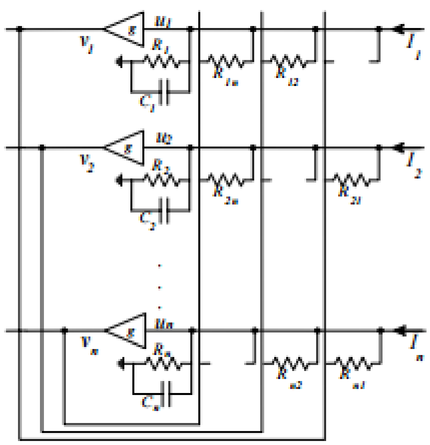

Fig 4: Understanding HANN with hardware configuration

Each artificial neuron in the network can be realized as a separate block with a non-linear activation function. As per Universal approximation theorem (Also called Cybenko theorem), we can choose any function with necessary parameters. But sigmoid functions are preferred since they impinge non-linearity in the system and satisfies an interesting property between itself and it's first order derivative as follows:

$$\frac{d}{dt} sig(t) = sig(t)(1 - sig(t))$$

Let us choose sigmoid function as the activation function. We define it mathematically as below:

$$v_i = g_i(u_i) = \frac{1}{1+e^{-a_i \cdot u_i}}$$ --------------(1)

In the above hardware realization, weight W_{ij} has been realized with the resistance R_{ij} such that,

$W_{ij} = 1/R_{ij}$.

Taking the external direct input (I_i) and feedback from the output into the account, the total input to the artificial neurons in the network is given by:

$$\sum_{j\neq i} \frac{v_j}{R_{ij}} + I_i \quad \text{where } R_{ij} \text{ is interconnection weight form neuron } i \text{ to neuron } j.$$ ------(2)

For every output Vi, we will have two levels defined as V^0i and V^1i depicting 0 and 1 respectively. The threshold value Ui is related to the output as follows:

$$if\left(\sum_{j\neq i} \frac{v_j}{R_{ij}} + I_i < u_i\right)\{v_i = v_i^0\}$$
$$\text{else}\{v_i = v_i^1\}$$ -------(3)

For the circuit presented in the figure 4 , the equation can be written as:

$$C_i \frac{du_i}{dt} = \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{v_j}{R_{ij}} - \frac{u_i}{R_i} + I_i, \quad i = 1,2,\ldots,N$$ -------------(4)

Where Ri is the equivalent resistence connected to the capacitor of the cell Ci.

For the network to be stable as per Lyapunov theorem, the energy function (E) can be given as

$$E = -\frac{1}{2}\sum_{\substack{i=1 \\ i\neq j}}^{N}\sum_{j=1}^{N} \frac{v_j v_i}{R_{ij}} - \sum_{i=1}^{N} I_i v_i$$ -------------(5)

[**Note**: Here we have assumed that the number of activation linkages are large enough]

The rate of change of energy with respect to the output is given by taking differential on both sides:

$$\frac{\partial E}{\partial v_i} = -\sum_{j=1}^{N} \frac{v_j}{R_{ij}} - \sum_{i=1}^{N} I_i$$ -------(6)

Combining equation (4) and (6),

$$C_i \frac{du_i}{dt} = -\frac{u_i}{R_i} - \frac{\partial E}{\partial v_i}$$ --------(7)

Under Lypunov stability of the system,putting

$du_i = dt = 0,$

We get,

$$\partial E = -\frac{u_i}{R_i}\partial v_i \qquad \text{------(8)}$$

Equation (8) defines the stability criteria of HANN. So we can say that for a stable HANN, the differential change in energy function should be negative or zero (If the threshold is 0 itself).

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