SHIV SHAKTI

International Journal of in Multidisciplinary and Academic Research (SSIJMAR)

Vol. 3, No. 4, August-September (ISSN 2278 – 5973)

THE MATHEMATICS OF FINANCE

Dhaniya Satish Kumar
Research Scholar, Sai Nath University (Jharkhand)
Dr. Gagandeep Kaur Gulati
Research Supervisor, Sai Nath University (Jharkhand)

Impact Factor = 3.133 (Scientific Journal Impact Factor Value for 2012 by Inno Space Scientific Journal Impact Factor)

Indexing:

"The concept of interest is one of the central ideas in finance. Individuals, as well as business organizations, frequently encounter situations that involve cash receipts or disbursements over several periods of time. When this happens, interest rates and interest payments become important and sometimes vital considerations".

Concept and Relevance

Time Value of Money (TVM) refers to the fact that the money received today is different in its worth from the money receivable at some other time in future. In other words, the same principle can be stated as that the money receivable in future is less valuable than the money received today. The English Proverb '**A bird in hand is worth two in the bush**', possibly gives the correct implications of the concept of TVM. Every individual or a firm definitely has a preference to receive money today against the money receivable tomorrow. The obvious reason for this preference for receiving the money today is that the rupee received today has a higher value than the rupee receivable in future. This preference for current money as against future money is known as the time preference for money or simply Time Value of Money.

There are several reasons for this preference for current money as follows:-

a. ***Future Uncertainties*****:** One of the reasons for preference for current money is that there is a certainty about it whereas the future money has an uncertainty. There may be an apprehension that other party (the creditor) may become insolvent or untraceable.

b. ***Preference for Present Consumption*****:** Besides certainly, every person also has a preference for present consumption, though this preference may be subjective and differ from one person to another. The present money may be required for some specific purpose.

c. ***Reinvestment Opportunities*****:** Both the individuals and the firm have preference for present money because they have reinvestment opportunities available to them. If they have got the money, they can invest this money to get further returns on this. This opportunity to get returns will not be available if the money is not invested now. The existence of reinvestment opportunities and the urge to earn a return by investing this current money seem to be the obvious reason for the time preference for money. This expected return which can be earned by investing the present money in fact the Time Value of Money.

Since, in most of the financial decisions, a finance manager has to deal with monies of different time periods, he is often required to adjust the cash flows for Time Value of Money. That is why the concept of Time Value of Money is often regarded as the central concept of the theory of finance. A finance manager may frequently encounter situations that involve cash flows (in terms of revenues generated), over number of years. A firm may raise funds today by issuing debentures on which interest will have to be paid for several years together with redemption amount in one or several installments. Sound and effective decision making in respect of these and so many other similar situations should be based upon the cash flows that are comparable. The

absolute cash flows of different time periods can be made comparable by applying the concept of Time Value of Money.

Therefore, the Time Value of Money is of crucial significance to any finance manager and become important and vital consideration while taking financial decisions. The concept of Time Value of Money helps in converting the different rupee amounts arising at different point of time into equivalent values of a particular point of time (present or any time in future).

On the other hand, the PV of future money may be defined as the value of that money if it was received today.

The relationship between the PV and the FV arises because of the existence of the interest rate and the time gap. The interest rate and the time gap between the present money and the future money in fact tie the PV and the FV together in a mathematical relationship as follows:

$$FV = PV \times (1 + r)^n$$

$$\text{and, } PV = FV / (1 + r)^n$$

where r = % rate of interest, and
n = time gap

Both concepts of FV and PV are applied in financial decisions making. It is already stated that the cash flows of different time periods can be made comparable either -

(i) by compounding the present money to a future date i.e. by finding out the FV of a present money, or
(ii) by discounting the future money to present date i.e. by finding out the PV of a future money.

These techniques of compounding and discounting as a tool to incorporate the TVM in the financial decision making have been discussed as follows:-

Compounding Technique

The compounding technique is used to find out the FV of present money. It is the same as the concept of compound interest, wherein the interest earned in a preceding year is reinvested at the prevailing rate of interest for the remaining period. Thus, the accumulated amount (principal + interest) at the end of a period becomes the principal amount for calculating the interest for the next period. The compounding technique to find out the FV of present money can be explained with reference to:

a. The FV of a single present cash flow, and
b. The FV of a series of cash flows.

***The FV of a single present cash flow*:** It is already seen that the FV may be defined in terms as follows:

$$FV = PV \times (1 + r)^n$$

$$\text{and, } PV = FV / (1 + r)^n$$

where r = % rate of interest, and
n = Time gap after which FV is to be ascertained.

The above equation explains that the FV depends upon the combination of three variables i.e. the PV, the r, and the n. if any one of these three variables changes, the FV will also change. There can be an almost infinite number of combinations of these three variables and therefore there can be corresponding infinite number of FVs.

***Future Value of a Series of Equal Cash flows or Annuity of Cash flows*:** Quite often a decision may result in the occurrence of cash flows of the same amount every year for a number of years consecutively, instead of single cash flows. An annuity is thus, a finite series of equal cash flows made at regular intervals.

In this case, each cash flow is to be compounded to find out its FV. The total of these FVs of all these cash flows will be the total FV of the annuity. The FV of an annuity also depends upon three variables i.e. the annual amount, the rate of interest and the time period. In order to find out the rate of an annuity, the pre-calculated mathematical table is available for various combinations of the rate of interest, r, and the time period, n.

Discounting Technique

After going through the process of determining the FV present money or a present series, now the process of finding out the Present Value (PV) of a future sum or a future series can be discussed. This process is in fact the reverse of compounding technique and is known as the discounting technique. As there are FVs of sums invested now, calculated as per the compounding techniques, there are also the present values of a cash flow scheduled to occur in future. The present value is calculated by discounting technique by applying Equation above.

***Present Value of a Future Sum*:** The present value of a future sum will be worth less than the future sum because one foregoes the opportunity to invest and thus foregoes the opportunity to earn interest during that period. Expectation of receiving the money in future means that the money is not available presently and therefore one has to forego the interest which could be earned, had the money been available now. This also makes a person to loose the opportunities to get reward/return in terms of interest earnings on that investment. This interest foregone is the cost to the investor and the future expected money must be adjusted for this cost. As the length of time for which one has to wait for the future money increases, the cost attached to delay also increases reflecting the compounded value of the lost opportunities. In order to find out the PV of future money, this opportunity cost of the money is to be deducted from the future money.

***The PV of a Series of Equal Future Cash flows or Annuity*:** A decision taken today may result in a series of future cash flows of the same amount over a period of number of years.

In addition to the types of cash flows discussed above, there can be some other types of cash flows also.

Perpetuity: Perpetuity may be defined as an infinite series of equal cash flows occurring at regular intervals. It has indefinitely long life. If a deposit of Rs.1,000 is made in a savings bank account at 4.5% for an indefinite period then the yearly interest of Rs. 50 is perpetuity of interest income so long as the initial deposit of Rs.1,000 is kept unchanged. In order to find out the PV of perpetuity, the present value of each of the infinite number of cash flows should be added. If the first occurrence of the perpetuity takes place after 1 year from today then the present value of the perpetuity may be calculated with the help of the following Equation:

$$PV = \frac{\text{Cash flow}}{(1+r)^1} + \frac{\text{Cash Flow}}{(1+r)^2} + \ldots\ldots\ldots + \frac{\text{Cash Flow}}{(1+r)}$$

Conceptually, it is difficult or rather impossible to find out the PV of perpetuity. However, mathematically it is the easiest stream of the cash flows to value. Mathematically, infinite summation adds up to the simple version.

$$PV_p = \text{Annual Cash flow}/r$$

Where PV_p is the present value of perpetuity; thus, the present value of perpetuity is equal to the amount of perpetuity divided by the rate of interest.

Annuity Due: The discussion on FV or the PV of an annuity was based on the presumption that the cash flows occur at the end of each of the periods starting from now. However, in practice the cash flows may also occur in the beginning of each period. Such a situation is known as annuity due.

In an ordinary annuity of n years, the first cash flow will occur after 1 year from now and the last cash flow will occur at the end of the n^{th} period. On the other hand, in annuity due, the first cash flow occurs now and the last cash flow will occur in the beginning of the n^{th} year.

Growing Perpetuity: A growing perpetuity may be defined as an infinite series of periodic cash flows which grow at a constant rate per period.

$$PV = \text{Cash flow} / (r - g)$$

where cash flow = the cash flow at the end of the first period,
r = rate of interest
and g = growth rate in perpetuity amount

However, it may be noted that above formula can be used only if the rate of interest is more than the rate of growth i.e. $r > g$.

***Growing Annuity*:** A growing annuity may be defined as a finite series of equal and periodic cash flows growing at a constant rate every period. Since, an annuity is nothing but a truncated growing perpetuity; the growing annuity can also be viewed

as a truncated perpetuity. Thus, the valuation of growing annuity is akin to the valuation of growing perpetuity.

Applications of the Concept of Time Value of Money

Finance Manager has to deal with varying situations of decision making where the concept of TVM needs to be applied in one form or the other. However, it may be noted that the proper understanding of the cash flows, selection of an appropriate discounting/compounding technique and applying the technique correctly are some of the prerequisites of an appropriate decision based on TVM. Practice and experience, both are required for the proper use of the techniques of TVM. The following are some of the applications of the concept of the TVM:

Finding out the implicit of interest: Several financial institutions have issued the Deep Discount Bonds (DDB) where the investor is required to pay a specific amount per bond at the time of issue and receives a much larger amount at the end of a specified period. The rate of interest however, is not given. The technique of TVM can be applied to find out the implicit rate of interest as applicable to DDBs.

A finance company may offer a scheme under which an investor may be required to deposit a specific amount now and to receive a series of returns for a specific number of years. The scheme may be acceptable to an investor only if the implicit rate of interest is more than the normal rate of interest.

Finding out the number of periods: Sometimes, one may be interested to find out the time over which a certain amount will grow at a given rate of interest to a certain value. In this case, the value of 'n' can be ascertained by solving Equation:

$$FV = PV\ (1 + r)^n$$

Sinking Funds: Quite often, one may be interested to accumulate a target amount over a given period inclusive of interest for the period in such a way that the annual amount being subscribed over the period is same for all years. In case of a business firm, a finance manager may be interested to accumulate a target amount in order to replace an asset or in order to repay a liability at the end of a specified period. In this case, the annual accumulation by the finance manager in fact becomes the annuity for a given period where each of the annual subscription/accumulation will be invested for the remaining period so that the total accumulation at the end of the given period is equal to the target amount.

Capital Recovery: Sometimes, one may be interested to find out the equal annual amount paid in order to redeem a loan of a specified amount over a specified period together with the interest at a given rate for that period.

Deferred Payments: Suppose a person takes a loan of a specified amount at a given rate of interest. He wants to repay this loan together with interest in such a way that the annual amount being paid is same and further that the first payment be made a few years from now. In this case, the interest for the period for which the payment has been delayed (i.e. the period from the date of loan to the date of first payment) should

also be considered in finding out the annual payment for the repayment of loan together with the interest.

Reference

- Harold Markowitz, *Portfolio Selection*, Journal of Finance, 7, 1952, pp. 77–91
- William Sharpe, *Investments*, Prentice-Hall, 1985
- Attilio Meucci, *P versus Q: Differences and Commonalities between the Two Areas of Quantitative Finance*, GARP Risk Professional, February 2011, pp. 41–44
- Nicole El Karoui, *The Future of Financial Mathematics*, ParisTech Review, September 2013